Ex: Obtain the block diagram representation of the system in cascade form for following transfer functions

(i) $$H(z) = \frac{4z^3 + 16z^2 + 4z - 24}{2z^4 + 1.6z^3 + 0.5z^2 + 0.1z}$$

Sol: To realise $H(z)$ in cascade form, we have to put $H(z)$ in the following form

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots}{1 + b_1 z^{-1} + b_2 z^{-2} + \cdots}$$

$$H(z) = a_0 \prod_{i=1}^{K} H_i(z)$$

where $H_i(z)$ may be a first order section of $\frac{1+a_1 z^{-1}}{1+b_1 z^{-1}}$

or second order section of $\frac{1 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}$

i) Multiply $N(z)$ and $D(z)$ with z^{-4}

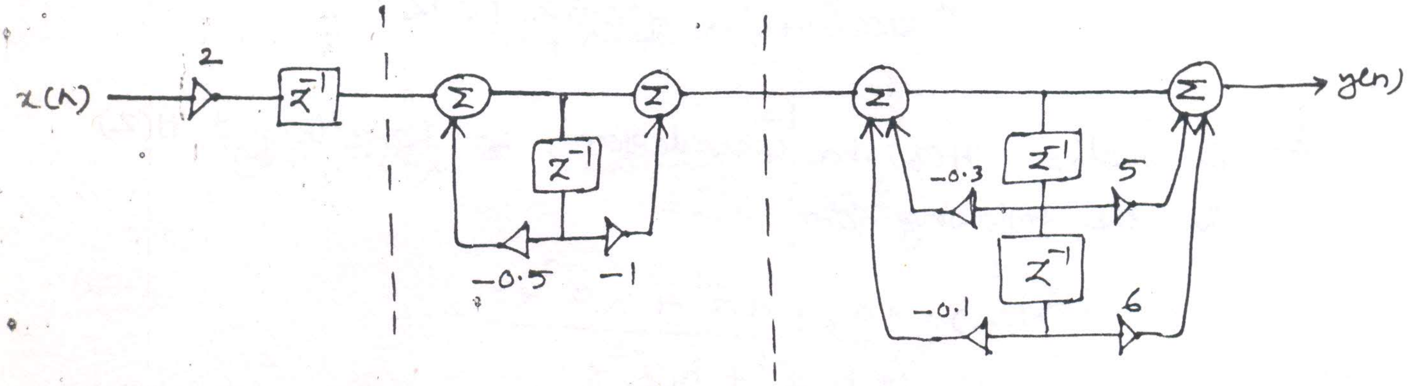
$$\therefore H(z) = \frac{4z^{-1} + 16z^{-2} + 4z^{-3} - 24z^{-4}}{2 + 1.6z^{-1} + 0.5z^{-2} + 0.1z^{-3}}$$

$$H(z) = \frac{2z^{-1} + 8z^{-2} + 2z^{-3} - 12z^{-4}}{1 + 0.8z^{-1} + 0.25z^{-2} + 0.05z^{-3}}$$

$$= \frac{2z^{-1}(1 + 4z^{-1} \overset{(+)}{\cancel{+}} z^{-2} - 6z^{-3})}{1 + 0.8z^{-1} + 0.25z^{-2} + 0.05z^{-3}}$$

$$= \frac{2z^{-1}(1 - z^{-1})(1 + 5z^{-1} + 6z^{-2})}{(1 + 0.5z^{-1})(1 + 0.3z^{-1} + 0.1z^{-2})}$$

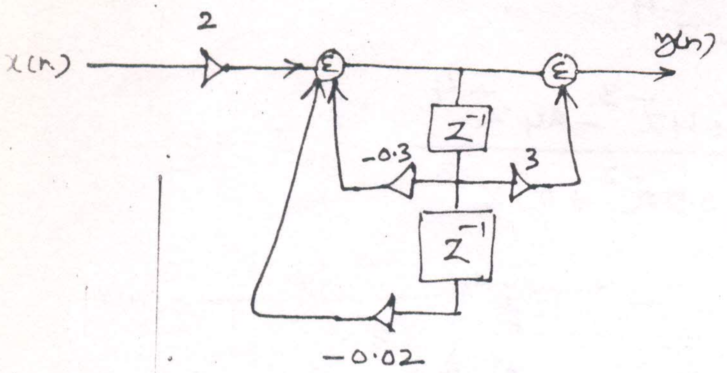
$$H(z) = \left[2z^{-1}\right]\left[\frac{1-z^{-1}}{1+0.5z^{-1}}\right]\left[\frac{1+5z^{-1}+6z^{-2}}{1+0.3z^{-1}+0.1z^{-2}}\right]$$

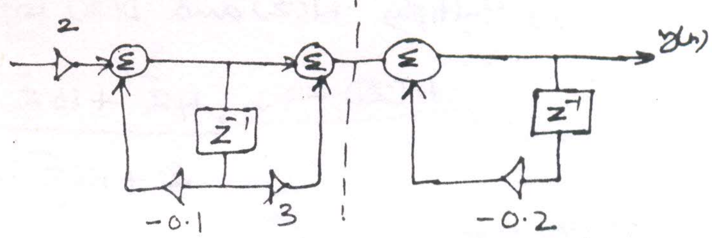


(ii)

$$H(z) = \frac{2z(z+3)}{z^2+0.3z+0.02} = \frac{2(z^2+3z)}{z^2+0.3z+0.02} = \frac{2(z^2+3z)}{(z+0.1)(z+0.2)}$$

$$H(z) = \frac{2(1+3z^{-1})}{1+0.3z^{-1}+0.02z^{-2}} \quad \text{or} \quad \frac{2(1+3z^{-1})}{(1+0.1z^{-1})(1+0.2z^{-1})}$$





(iii)

$$H(z) = \frac{z^{-1} + 4z^{-2}}{5 - 2z^{-1} + 0.15\, z^{-2}}$$

$$H(z) = \frac{z^{-1}(1 + 4z^{-1})}{5 - 2z^{-1} + 0.15 z^{-2}} = \frac{0.2 z^{-1}(1 + 4z^{-1})}{1 - 0.4z^{-1} + \underset{0.03}{\cancel{0.3}} z^{-2}}$$

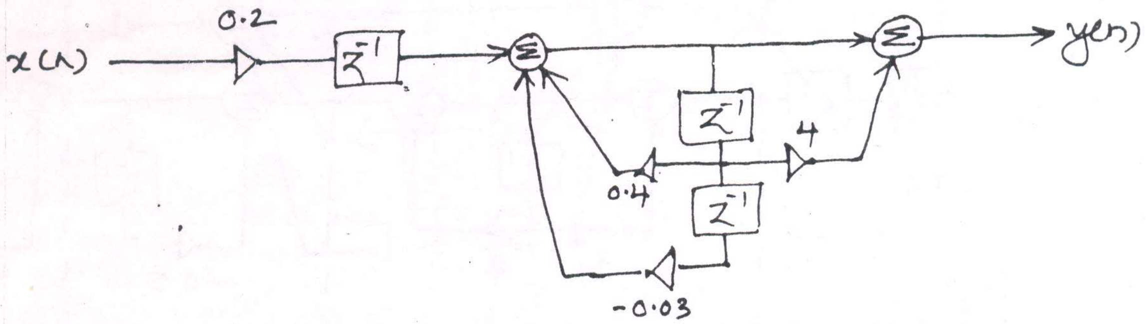
$$H(z) = (0.2)(z^{-1}) \left[\frac{1 + 4z^{-1}}{1 - 0.4z^{-1} + \underset{0.03}{\cancel{0.3}} z^{-2}} \right] \quad \text{——— ①}$$

(or)

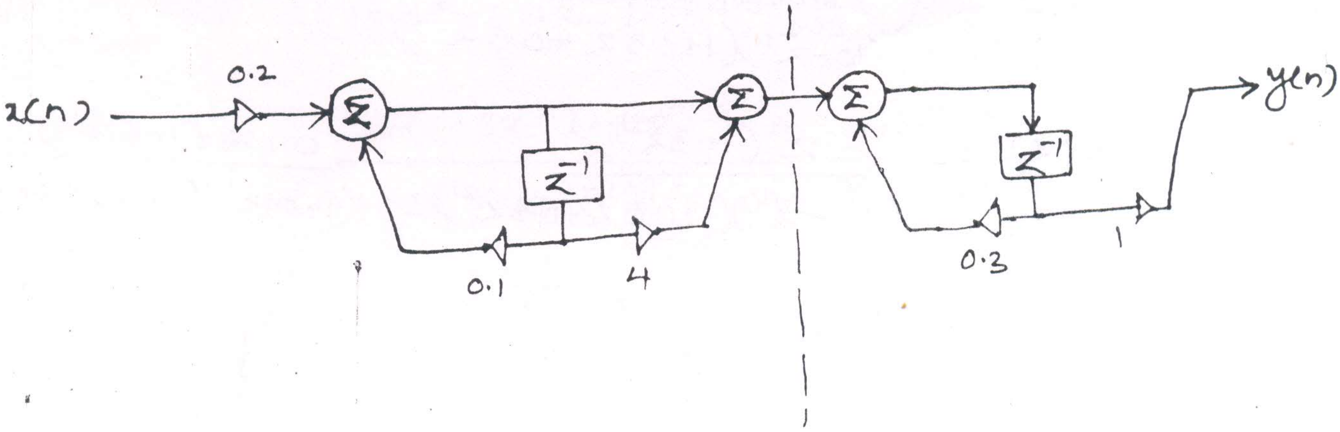
$$H(z) = (0.2)(z^{-1}) \frac{1 + 4z^{-1}}{(1 - 0.1z^{-1})(1 - 0.3z^{-1})} \quad \text{——— ②} = 0.2 \left(\frac{1+4z^{-1}}{1-0.1z^{-1}}\right)\left(\frac{z^{-1}}{1-0.3z^{-1}}\right)$$

(using 1st order sections)

For eqn ①:



For eqn ②:

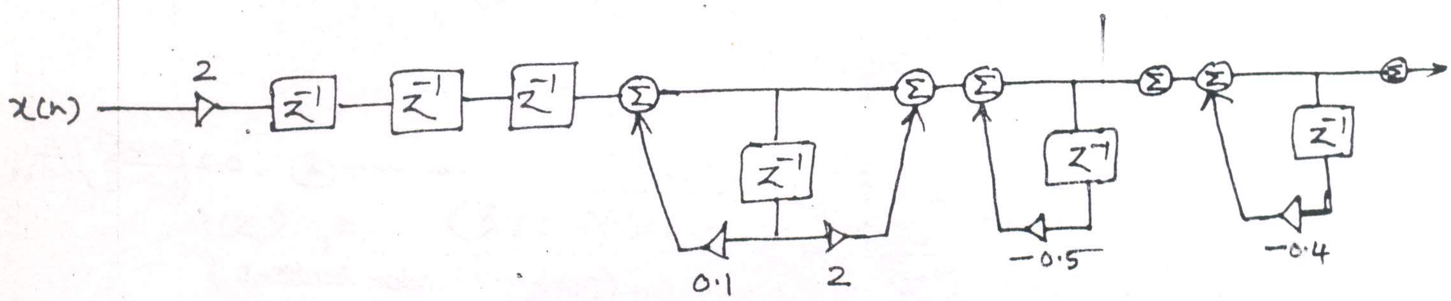


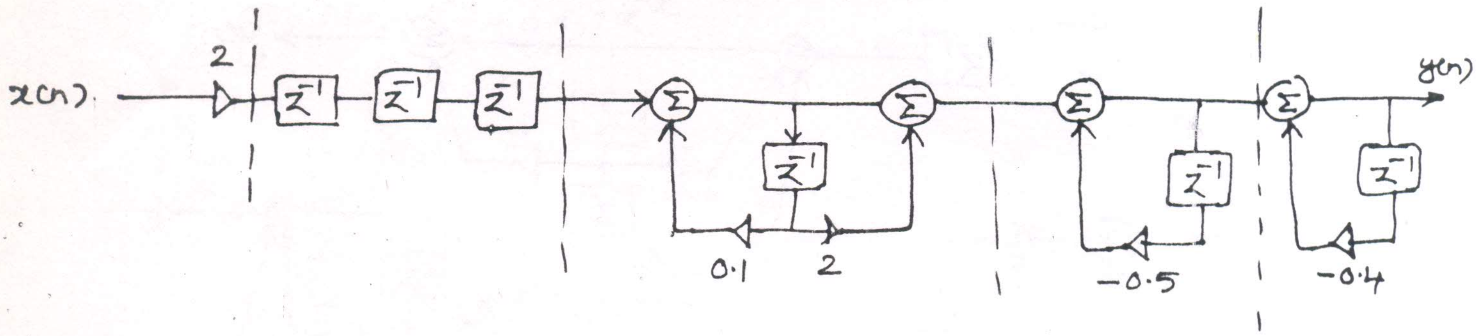
(iv)

$$H(z) = \frac{2(z+2)}{z(z-0.1)(z+0.5)(z+0.4)} \quad \text{using FOS}$$

$$H(z) = \frac{2z^{-4}(z+2)}{1(1-0.1z^{-1})(1+0.5z^{-1})(1+0.4z^{-1})}$$

$$H(z) = \frac{2z^{-3}(1+2z^{-1}) \cdot (1) \cdot (1)}{(1-0.1z^{-1})\,(1+0.5z^{-1})\,(1+0.4z^{-1})}$$



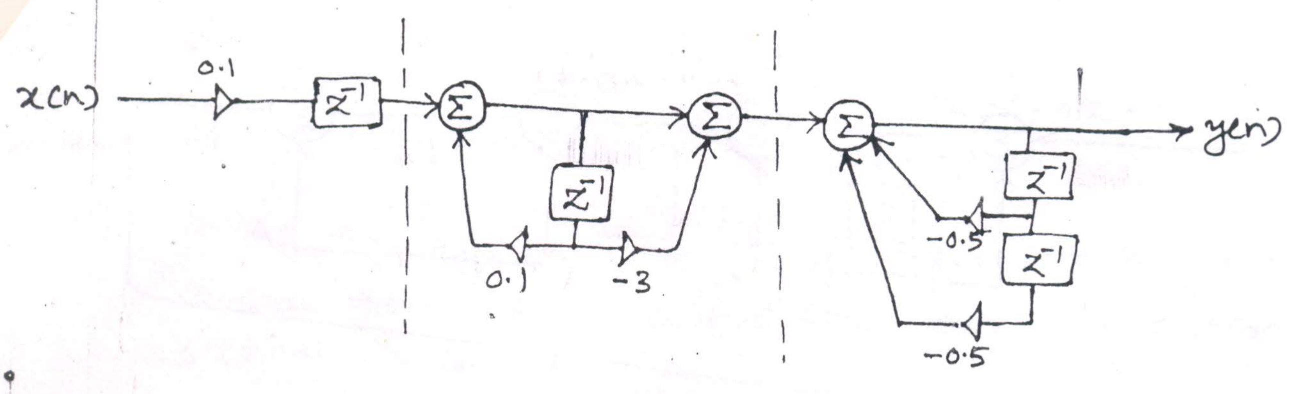


(v)

$$H(z) = \frac{z^{-1} - 3z^{-2}}{(10-z^{-1})(1+0.5z^{-1}+0.5z^{-2})}$$

$$H(z) = \frac{z^{-1}(1-3z^{-1})}{(10-z^{-1})(1+0.5z^{-1}+0.5z^{-2})} = \frac{0.1z^{-1}(1-3z^{-1})}{(1-0.1z^{-1})(1+0.5z^{-1}+0.5z^{-2})}$$

$$H(z) = (0.1)(z^{-1})\left[\frac{1-3z^{-1}}{1-0.1z^{-1}}\right]\left[\frac{1}{1+0.5z^{-1}+0.5z^{-2}}\right]$$

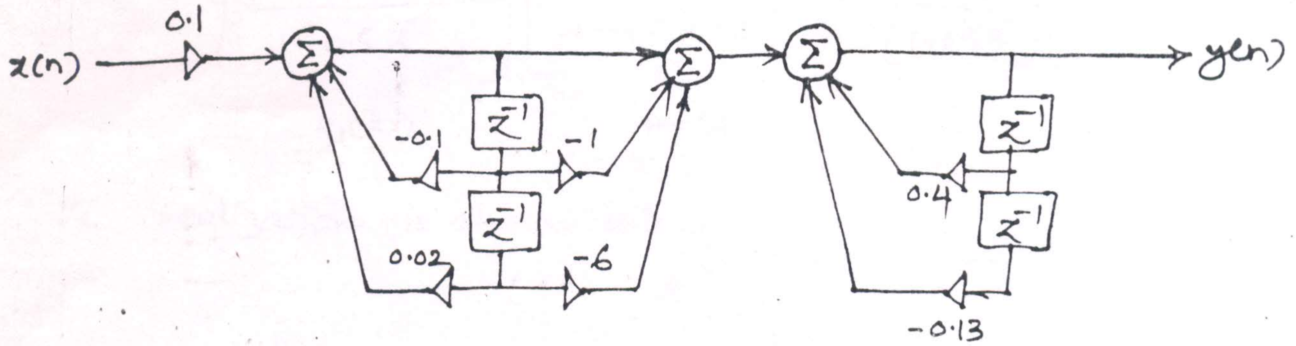


(vi)

$$H(z) = \frac{1 - z^{-1} - 6z^{-2}}{(10 + z^{-1} - 0.2z^{-2})(1 - 0.4z^{-1} + 0.13z^{-2})}$$

$$= \frac{0.1(1 - z^{-1} - 6z^{-2})}{(1 + 0.1z^{-1} - 0.02z^{-2})} \cdot \frac{1}{(1 - 0.4z^{-1} + 0.13z^{-2})}$$

The implementation using second order sections is shown below



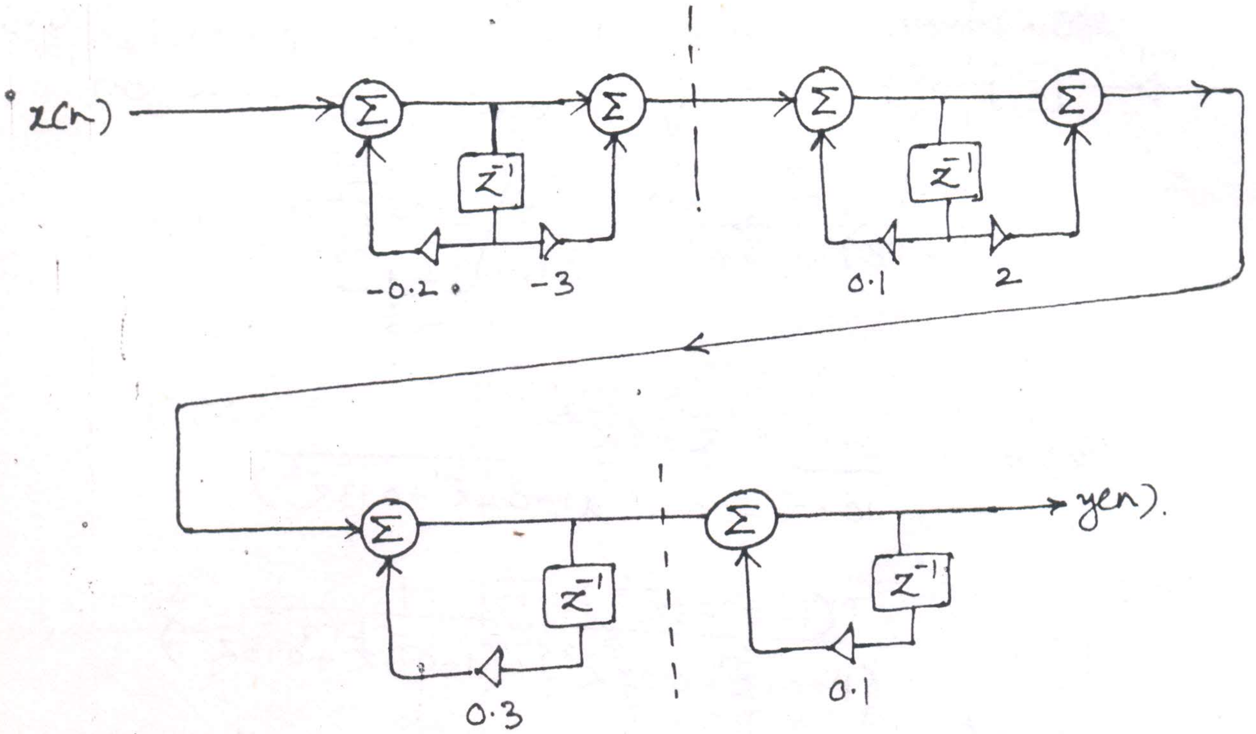
(vii)

$$H(z) = \frac{(1 - z^{-1} - 6z^{-2})}{(1 + 0.1z^{-1} - 0.02z^{-2})(1 - 0.4z^{-1} + 0.03z^{-2})}$$

Realize $H(z)$ using First Order Sections in cascade form.

$$H(z) = \frac{(1 - 3z^{-1})(1 + 2z^{-1})}{[(1 + 0.2z^{-1})(1 - 0.1z^{-1})][(1 - 0.3z^{-1})(1 - 0.1z^{-1})]}$$

$$H(z) = \left[\frac{1 - 3z^{-1}}{1 + 0.2z^{-1}}\right]\left[\frac{1 + 2z^{-1}}{1 - 0.1z^{-1}}\right]\left[\frac{1}{1 - 0.3z^{-1}}\right]\left[\frac{1}{1 - 0.1z^{-1}}\right]$$



x(n)
Σ
z⁻¹
-0.2
-3
0.1
2
0.3
y(n).

Ex: Given the system function $H(z) = \dfrac{(z+1)^2}{(4z^3 - 2z^2 + 1)}$ obtain the cascade form of realization.

Sol: As the system is of third order, we can have cascade form with a first order section and second order section in cascade.

$$H(z) = \frac{(z+1)^2}{4z^3 - 2z^2 + 1}$$

$D(z)$:
if $z = -\frac{1}{2} \Rightarrow D(z) = 0$
$\therefore (z + \frac{1}{2})$ is one of the factors.
Using synthetic division

$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & -2 & 0 & 1 \\ & 0 & -2 & 2 & -1 \\ \hline & 4 & -4 & 2 & \boxed{0} \end{array}$$

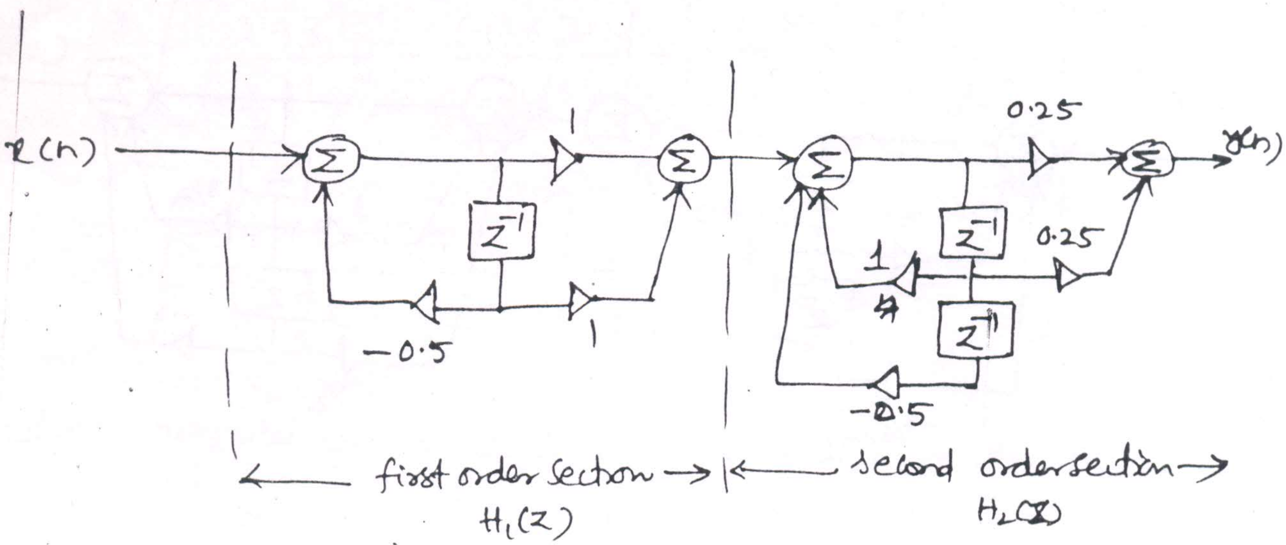
$\therefore D(z) = (z+\frac{1}{2})(4z^2 - 4z + 2)$

$$H(z) = \frac{(z+1)(z+1)}{(z+0.5)(4z^2-4z+2)}$$

$$H(z) = \left[\frac{z+1}{z+0.5}\right]\left[\frac{z+1}{4z^2-4z+2}\right]$$

or $$H(z) = \underbrace{\left[\frac{1+z^{-1}}{1+0.5z^{-1}}\right]}_{H_1(z)}\underbrace{\left[\frac{1+z^{-1}}{4-4z^{-1}+2z^{-2}}\right]}_{H_2(z)} = \left[\frac{1+z^{-1}}{1+0.5z^{-1}}\right]\left[\frac{0.25+0.25z^{-1}}{1-z^{-1}+0.5z^{-2}}\right]$$

The realization is shown below:



Ex: Determine the cascade and parallel realizations for the system described by the transfer function

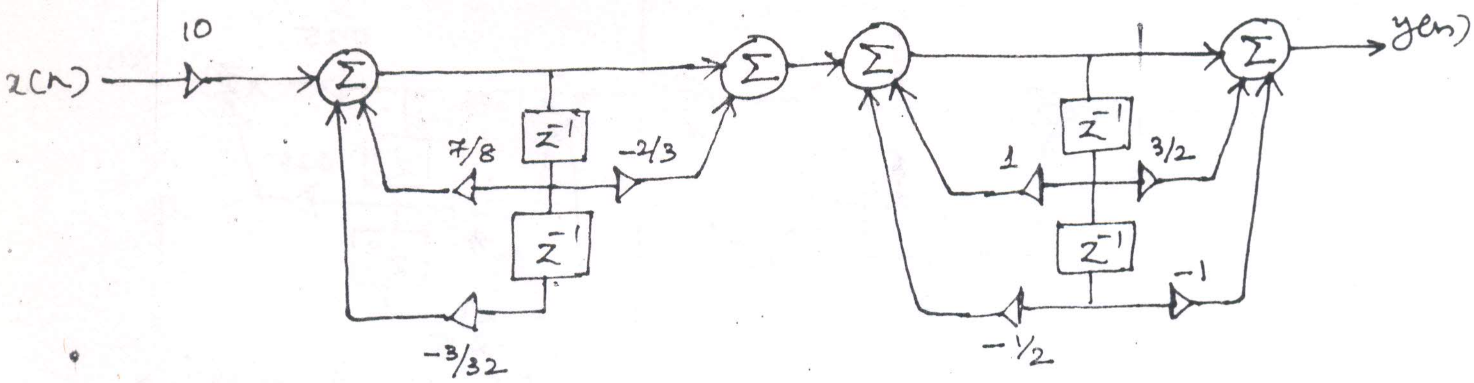
$$H(z) = \frac{10(1-\frac{1}{2}z^{-1})(1-\frac{2}{3}z^{-1})(1+2z^{-1})}{(1-\frac{3}{4}z^{-1})(1-\frac{1}{8}z^{-1})[1-(\frac{1}{2}+j\frac{1}{2})z^{-1}][1-(\frac{1}{2}-j\frac{1}{2})z^{-1}]}$$

Sol: Cascade: one possible pairing of poles is

$$H(z) = \frac{10(1-\frac{2}{3}z^{-1})(1+\frac{3}{2}z^{-1}-z^{-2})}{(1-\frac{7}{8}z^{-1}+\frac{3}{32}z^{-2})(1-z^{-1}+\frac{1}{2}z^{-2})}$$

$$H(z) = 10\cdot\left[\frac{1-\frac{2}{3}z^{-1}}{1-\frac{7}{8}z^{-1}+\frac{3}{32}z^{-2}}\right]\left[\frac{1+\frac{3}{2}z^{-1}-z^{-2}}{1-z^{-1}+\frac{1}{2}z^{-2}}\right]$$

$H(z) = 10\cdot[H_1(z)]\cdot[H_2(z)]$ This realization is depicted in the below fig:



To obtain parallel form realization, $H(z)$ must be put in PFE form (As the $H(z)$ is in terms of z^{-1}, donot try for $\frac{H(z)}{z}$ PFE)

$$H(z) = \frac{A_1}{1-\frac{3}{4}z^{-1}} + \frac{A_2}{1-\frac{1}{8}z^{-1}} + \frac{A_3}{\left[1-\left(\frac{1}{2}+j\frac{1}{2}\right)z^{-1}\right]} + \frac{A_4}{\left[1-\left(\frac{1}{2}-j\frac{1}{2}\right)z^{-1}\right]}$$

Note if poles are complex conjugate, then the coefficents in the PFE are also complex conjugate

(or)

complex conjugate poles result in complex conjugate coefficients in the PFE.

i.e., $A_4 = A_3^*$

After some arithmetic, we find that

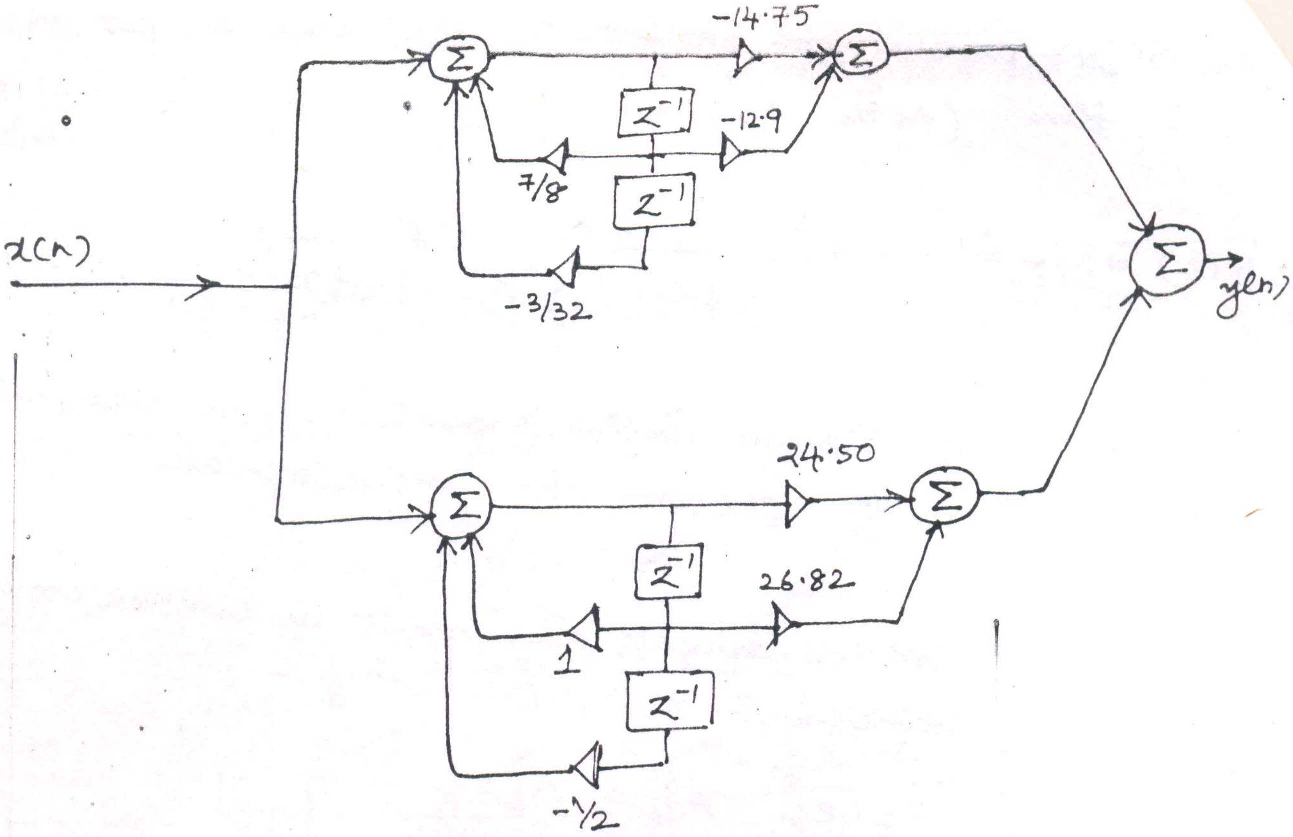
$A_1 = 2.93$; $A_2 = -17.68$

$A_3 = 12.25 - j14.57$ and $A_4 = 12.25 + j14.57$.

$$H(z) = \left[\frac{2.93}{1-\frac{3}{4}z^{-1}} - \frac{(-17.6)}{(1-\frac{1}{8}z^{-1}}\right] + \left\{\frac{12.25-j14.57}{\left[1-\left(\frac{1}{2}+j\frac{1}{2}\right)z^{-1}\right]}\right\} + \left\{\frac{12.25+j14.57}{\left[1-\left(\frac{1}{2}-j\frac{1}{2}\right)z^{-1}\right]}\right\}$$

$$= \left[\frac{-14.75-12.90z^{-1}}{1-\frac{7}{8}z+\frac{3}{32}z^{-2}}\right] + \left[\frac{24.50+26.82z^{-1}}{1-z^{-1}+\frac{1}{2}z^{-2}}\right]$$

whose parallel form realization is illustrated in the fig below

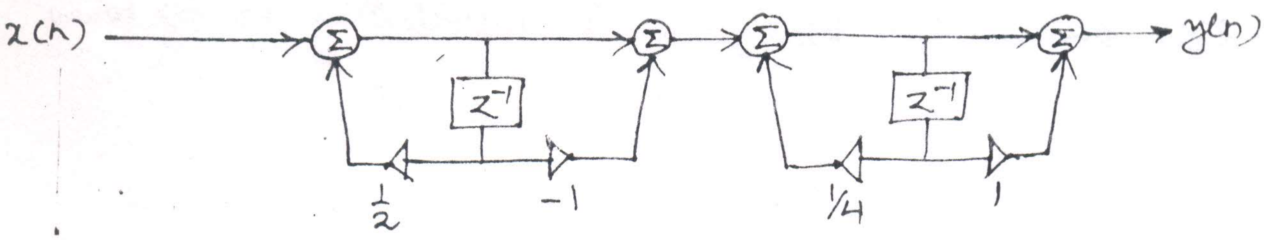


Ex: Realize the following transfer function as cascade and parallel form using first order sections

$$H(z) = \frac{(1-z^{-1})^2}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

Sol: <u>for cascade</u>

$$H(z) = \frac{(1-z^{-1})}{(1-\frac{1}{2}z^{-1})} \cdot \frac{(1-z^{-1})}{(1-\frac{1}{4}z^{-1})} = H_1(z) \cdot H_2(z)$$



for Parallel realizations: we find PFE for $X(Z)$

$$X(Z) = \frac{(Z-1)^2}{(Z-\frac{1}{2})(Z-\frac{1}{4})}$$

$$\therefore \frac{X(Z)}{Z} = \frac{(Z-1)^2}{Z(Z-\frac{1}{2})(Z-\frac{1}{4})}$$

or $$\frac{X(Z)}{Z} = \frac{A}{Z} + \frac{B}{(Z-\frac{1}{2})} + \frac{C}{(Z-\frac{1}{4})}$$

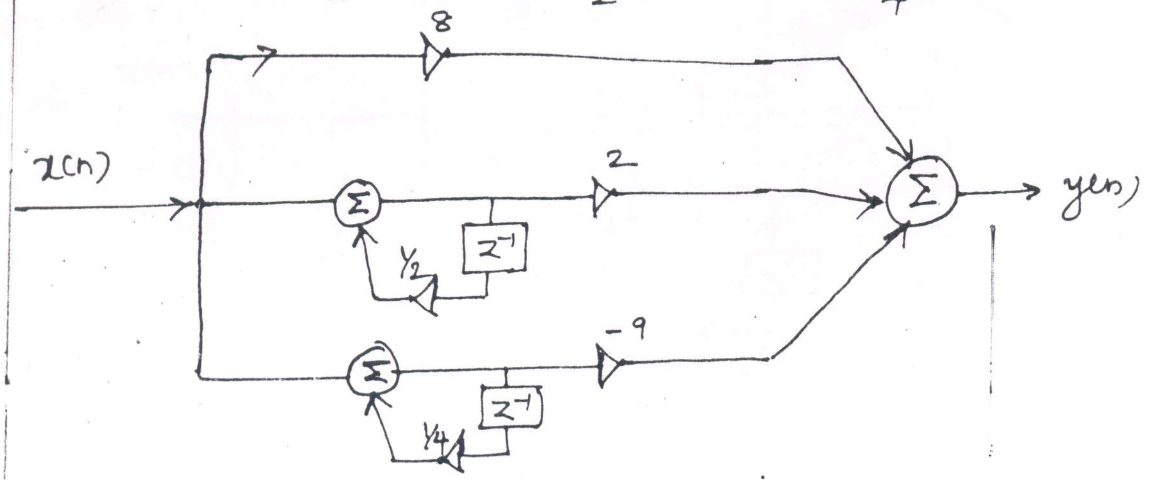
where $$A = Z\cdot\frac{X(Z)}{Z}\Big|_{Z=0} = \frac{1}{-\frac{1}{2}\times-\frac{1}{4}} = 8$$

$$B = (Z-\tfrac{1}{2})\frac{X(Z)}{Z}\Big|_{Z=\frac{1}{2}} = \frac{1/4}{\frac{1}{2}\times\frac{1}{4}} = 2$$

$$C = (Z-\tfrac{1}{4})\frac{X(Z)}{Z}\Big|_{Z=\frac{1}{4}} = \frac{9/16}{\frac{1}{4}\cdot(-\frac{1}{4})} = -9$$

$$\therefore X(Z) = 8 + \frac{2Z}{Z-\frac{1}{2}} + \frac{-9Z}{Z-\frac{1}{4}}$$

$X(Z) = 8 + \dfrac{2}{1-\frac{1}{2}Z^{-1}} + \dfrac{-9}{1-\frac{1}{4}Z^{-1}}$; The realization is



Ex: Obtain the parallel realization of the system with transfer function

$$H(z) = \frac{4z^3+16z^2+4z-24}{2z^4+1.6z^3+0.5z^2+0.1z}$$

$$= \frac{4z^3+16z^2+4z-24}{2z^4+1.6z^3+0.5z^2+0.1z} = \frac{4z^3+16z^2+4z-24}{z(2z^3+1.6z^2+0.5z+0.1)}$$

$$= \frac{2z^3+8z^2-2z-12}{z(z^3+0.8z^2+0.25z+0.05)} = \frac{2z^3+8z^2-2z-12}{z(z+0.5)(z^2+0.3z+0.1)}$$

For parallel realization we must obtain the partial fraction expansion of $\frac{H(z)}{z}$

$$\therefore \frac{H(z)}{z} = \frac{2z^3+8z^2-2z-12}{(z^2)(z+0.5)(z^2+0.3z+0.1)} = \frac{A_2}{z^2} + \frac{A_1}{z} + \frac{B}{(z+0.5)} + \frac{Cz+D}{(z^2+0.3z+0.1)}$$

obtain parallel realizations of the following transfer functions

(i) $$H(Z) = \frac{2Z(Z+3)}{Z^2+0.3Z+0.02}$$

$$H(Z) = \frac{2Z(Z+3)}{(Z+0.1)(Z+0.2)} \Rightarrow \frac{H(Z)}{Z} = \frac{2(Z+3)}{(Z+0.1)(Z+0.2)}$$

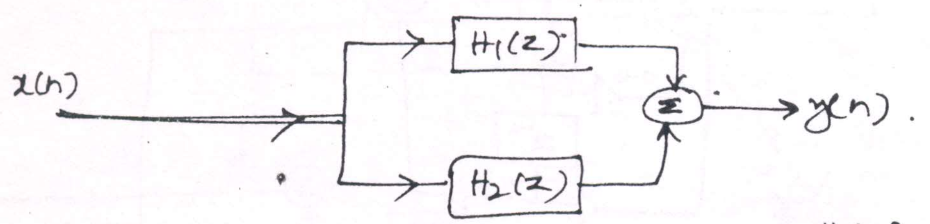
$$\frac{H(Z)}{Z} = \frac{A}{Z+0.1} + \frac{B}{Z+0.2}$$

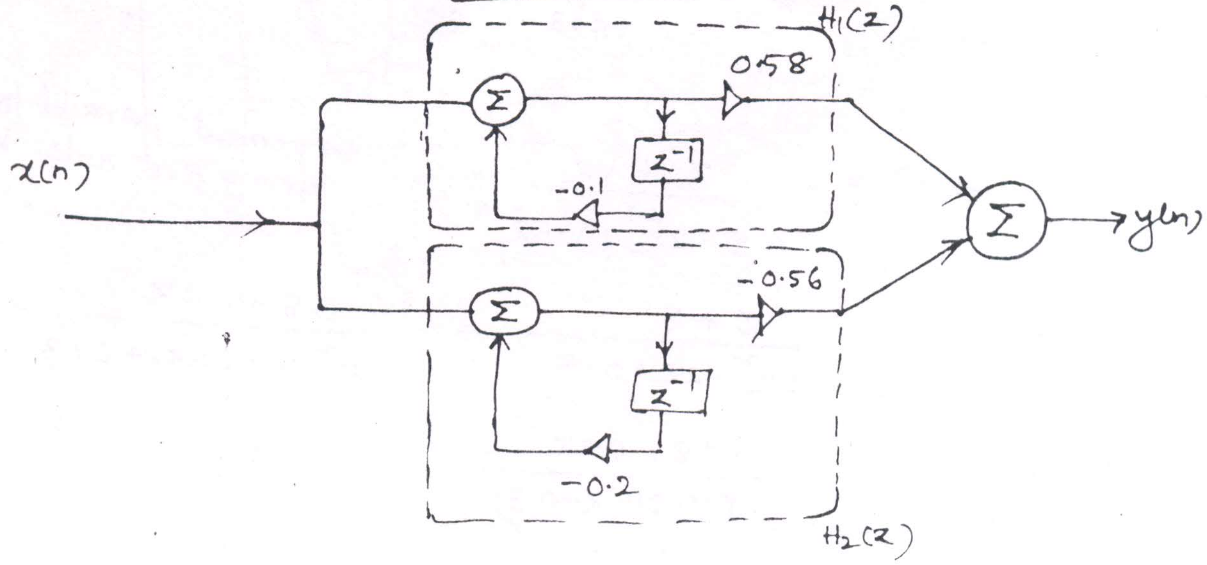
where

$$A = (Z+0.1)\frac{H(Z)}{Z}\Big|_{Z=-0.1} = \frac{2(-0.1+3)}{(-0.1+0.2)} = \frac{2\times 2.9}{0.1} = \underline{\underline{0.58}}$$

$$B = (Z+0.2)\frac{H(Z)}{Z}\Big|_{Z=-0.2} = \frac{2(-0.2+3)}{-0.2+0.1} = \frac{2\times 2.8}{-0.1} = \underline{\underline{-0.56}}$$

$$\therefore H(Z) = \frac{0.58Z}{Z+0.1} - \frac{0.56Z}{Z+0.2} = \underbrace{\frac{0.58}{1+0.1Z^{-1}}}_{H_1(Z)} + \underbrace{\frac{(-0.56)}{1+0.2Z^{-1}}}_{H_2(Z)}$$



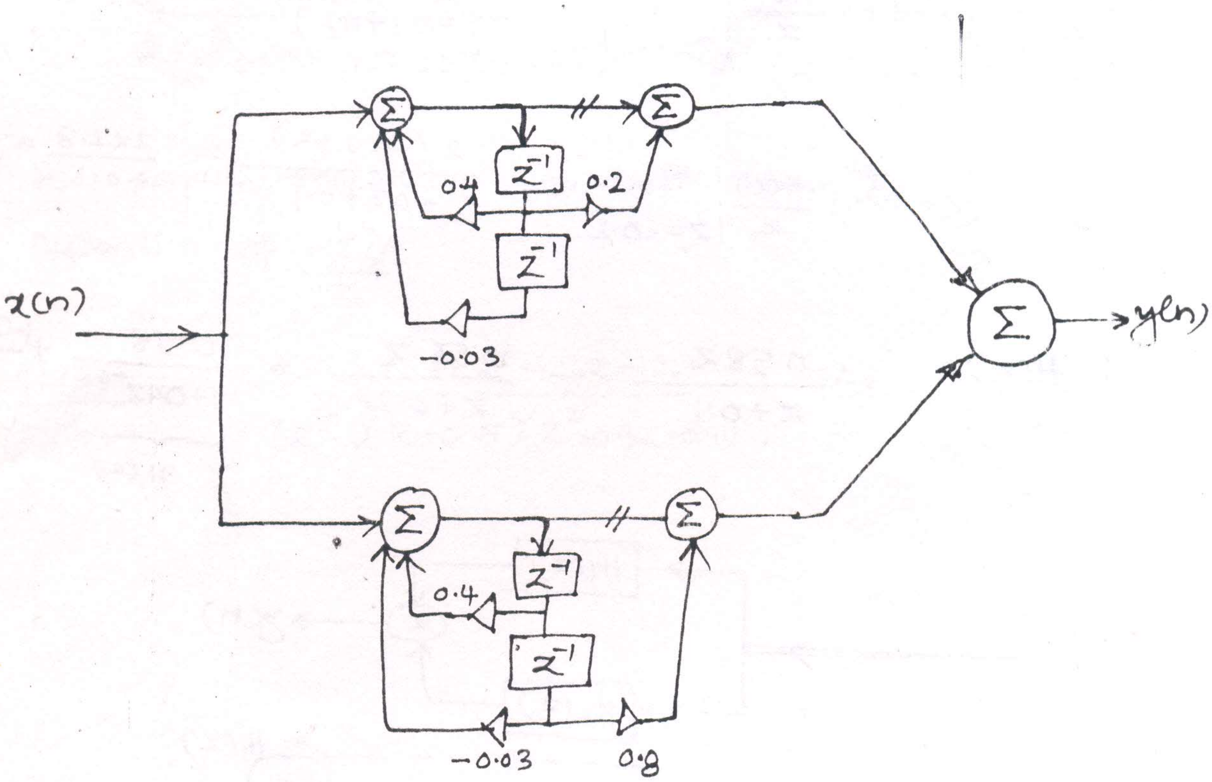


(ii)

$$H(z) = \frac{z^{-1} + 4z^{-2}}{5 - 2z^{-1} + 0.15z^{-2}}$$

$$H(z) = \frac{z^{-1}}{5 - 2z^{-1} + 0.15z^{-2}} + \frac{4z^{-2}}{5 - 2z^{-1} + 0.15z^{-2}}$$

$$H(z) = \underbrace{\frac{0.2z^{-1}}{1 - 0.4z^{-1} + 0.03z^{-2}}}_{H_1(z)} + \underbrace{\frac{0.8z^{-2}}{1 - 0.4z^{-1} + 0.03z^{-2}}}_{H_2(z)}$$



(III)

$$H(z) = \frac{z^{-1} + 4z^{-2}}{5 - 2z^{-1} + 0.15z^{-2}}$$: Implement parallel form using first order sections.

$$H(z) = \frac{z + 4}{5z^2 - 2z + 0.15} = \frac{0.8 + 0.2z}{z^2 + 0.04z + 0.03} = \frac{0.8 + 0.2z}{(z - 0.1)(z - 0.3)}$$

$$\frac{H(z)}{z} = \frac{0.8 + 0.2z}{(z)(z - 0.1)(z - 0.3)} = \frac{A}{z} + \frac{B}{z - 0.1} + \frac{C}{z - 0.3}$$

where $A = z \cdot \frac{H(z)}{z}\Big|_{z=0} = \frac{0.8+0.2z}{(z-0.1)(z-0.3)}\Big|_{z=0} = \frac{0.8}{0.03} = 26.66$

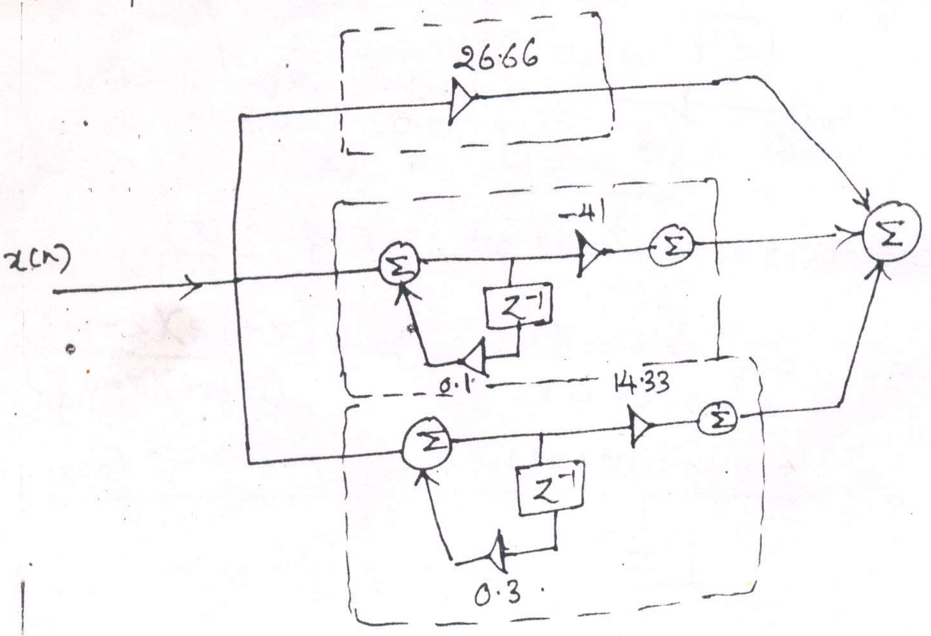
$$B = \frac{0.8+0.2z}{z(z-0.3)}\Big|_{z=0.1} = \frac{0.8+0.02}{(0.1)(-0.2)} = \frac{0.82}{-0.02} = -41$$

$$C = \frac{0.8+0.2z}{z(z-0.1)}\Big|_{z=0.3} = \frac{0.8+0.06}{(0.3)(0.2)} = \frac{0.86}{0.06} = 14.33$$

$$\therefore H(z) = 26.66 + \frac{(-41)z}{(z-0.1)} + \frac{(14.33)z}{z-0.3}$$

$$H(z) = 26.66 + \frac{(-41)}{1-0.1z^{-1}} + \frac{14.33}{1-0.3z^{-1}}$$

The parallel form realization is shown below



Ex: For the given $H(Z)$ develope the cascade and parallel realization schemes using first order sections:

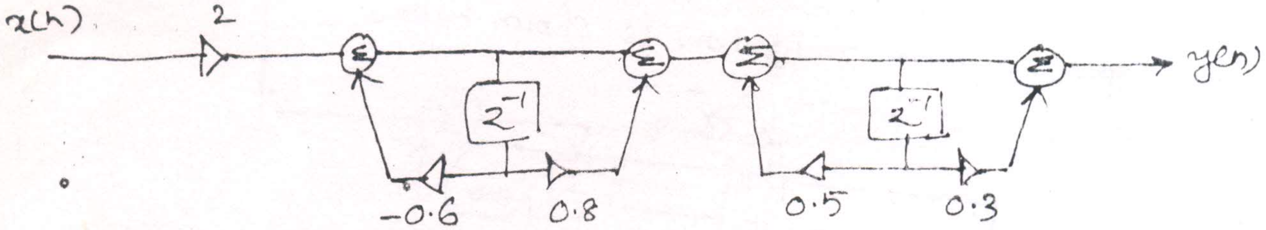
$$H(z) = \frac{2 + 2.2z^{-1} + 0.48z^{-2}}{1 + 0.1z^{-1} - 0.3z^{-2}}$$

Sol: For cascade form: $H(z) = H_1(z) \cdot H_2(z) \cdots$

$$H(z) = \frac{2(1 + 1.1z^{-1} + 0.24z^{-1})}{1 + 0.1z^{-1} - 0.3z^{-2}} = \frac{2(1 + 0.8z^{-1})(1 + 0.3z^{-1})}{1 + 0.1z^{-1} - 0.3z^{-2}}$$

$$= \frac{2(1 + 0.8z^{-1})(1 + 0.3z^{-1})}{(1 + 0.6z^{-1})(1 - 0.5z^{-1})}$$

$$= 2 \cdot \left[\frac{1 + 0.8z^{-1}}{1 + 0.6z^{-1}}\right]\left[\frac{1 + 0.3z^{-1}}{1 - 0.5z^{-1}}\right]$$



For parallel realization; we should get PFE of $\frac{X(z)}{z}$

$$\therefore H(z) = \frac{2z^2 + 2.2z + 0.48}{z^2 + 0.1z - 0.3} = \frac{2(z + 0.8)(z + 0.3)}{(z + 0.6)(z - 0.5)}$$

$$\frac{H(z)}{z} = \frac{2(z + 0.8)(z + 0.3)}{}$$