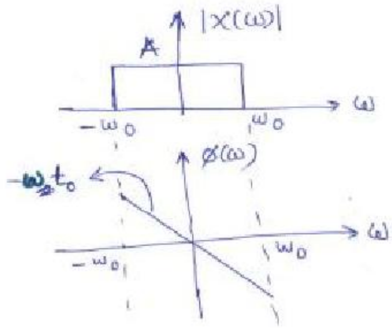
INVERSE FT FROM SPECTRA

Find IFT of the spectra shown.



Sol: We can express $X(\omega)$ as

$$X(\omega) = |X(\omega)|\, e^{j\angle X(\omega)}$$

$$= |X(\omega)|\, e^{j\phi(\omega)}$$

$$X(\omega) = A \cdot e^{j(-\omega t_0)} = A\, e^{-j\omega t_0}$$

$$\therefore \quad x(t) = \text{IFT}\left[X(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} A \cdot e^{-j\omega t_0} \cdot e^{j\omega t} \cdot d\omega = \frac{A}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega(t-t_0)}\, d\omega$$

$$= \frac{A}{2\pi}\left[\frac{e^{j\omega(t-t_0)}}{j(t-t_0)}\right]_{-\omega_0}^{\omega_0} = \frac{A}{2\pi j(t-t_0)}\left[e^{j\omega_0(t-t_0)} - e^{-j\omega_0(t-t_0)}\right]$$

$$= \frac{A}{\pi(t-t_0)} \times \left[\frac{e^{j\omega_0(t-t_0)} - e^{-j\omega_0(t-t_0)}}{2j}\right]$$

$$x(t) = \frac{A}{\pi(t-t_0)} \cdot \sin \omega_0(t-t_0)$$

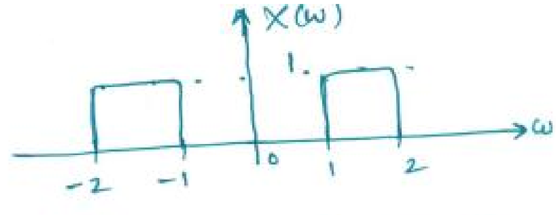
or

$$x(t) = \frac{A\cdot\omega_0}{\pi} \cdot \frac{\sin\left[\omega_0(t-t_0)\right]}{\left[\omega_0(t-t_0)\right]}$$

$$x(t) = \frac{A\omega_0}{\pi} \cdot \text{sinc}\left[\omega_0(t-t_0)\right]$$

$$x(t) = \frac{A\omega_0}{\pi}\, Sa\left[\omega_0(t-t_0)\right]$$

✓ Find the IFT of the ~~spectrum~~ spectrum shown.



Sol:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\cdot d\omega = \frac{1}{2\pi}\left[\int_{-2}^{-1} 1\cdot e^{j\omega t} d\omega + \int_{1}^{2} e^{j\omega t}\cdot d\omega\right]$$

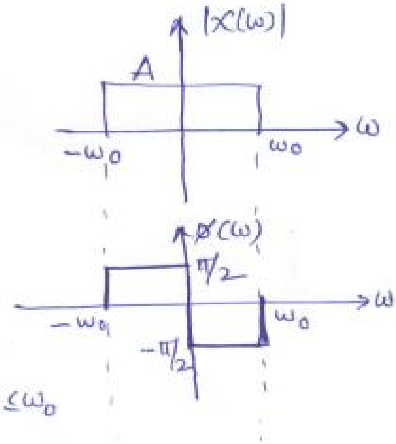
$$= \frac{1}{2\pi}\left[\int_{1}^{2} e^{-j\omega t}\cdot d\omega + \int_{1}^{2} e^{j\omega t}\cdot d\omega\right] = \frac{1}{2\pi}\left[\frac{e^{-j\omega t}}{-jt} + \frac{e^{j\omega t}}{jt}\right]_{1}^{2}$$

$$= \frac{1}{2\pi j t}\left[-e^{-j2t} + e^{-jt} + e^{j2t} - e^{jt}\right] = \frac{1}{2\pi j t}\left[\left(e^{j2t} - e^{-j2t}\right) - \left(e^{jt} - e^{-jt}\right)\right]$$

$$= \frac{1}{\pi t}\left[\frac{e^{j2t} - e^{-j2t}}{2j} - \frac{e^{jt} - e^{-jt}}{2j}\right] = \frac{1}{\pi t}\left[\sin(2t) - \sin(t)\right]$$

✓ Find the IFT of the spectrum shown

✓ Find the IFT of $X(\omega)$ for the spectra shown.



Sol: $X(\omega)$ can be expressed as

$$X(\omega) = |X(\omega)|\, e^{j\angle X(\omega)}$$

$$X(\omega) = |X(\omega)|\, e^{j\phi(\omega)} = A \cdot \begin{cases} e^{-j\pi/2} & ; 0 \le \omega \le \omega_0 \\ e^{j\pi/2} & ; -\omega_0 \le \omega \le 0 . \end{cases}$$

$$X(\omega) = \begin{cases} A \cdot e^{-j\pi/2} & ; 0 \le \omega \le \omega_0 \\ A \cdot e^{j\pi/2} & ; -\omega_0 \le \omega \le 0 \end{cases}$$

we know $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t} . d\omega = \frac{1}{2\pi}\left[\int_{-\omega_0}^{0} X(\omega) . e^{j\omega t} . d\omega + \int_{0}^{\omega_0} X(\omega) e^{j\omega t} d\omega\right]$

$$x(t) = \frac{1}{2\pi}\left[\int_{-\omega_0}^{0} A e^{j\pi/2} \cdot e^{j\omega t} . d\omega + \int_{0}^{\omega_0} A e^{-j\pi/2} \cdot e^{j\omega t} . d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{0}^{\omega_0} A \cdot j \cdot e^{-j\omega t} . d\omega + \int_{0}^{\omega_0} A(-j) \cdot e^{j\omega t} . d\omega\right]$$

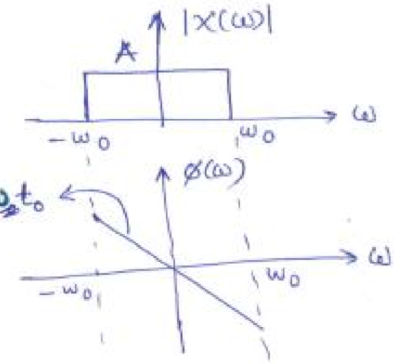
$$= \frac{Aj}{2\pi}\left[\frac{e^{-j\omega t}}{-jt} - \frac{e^{j\omega t}}{jt}\right]_0^{\omega_0} = \frac{Aj}{2\pi\, jt}\left[-e^{-j\omega_0 t} + 1 - e^{j\omega_0 t} + 1\right]$$

$$= \frac{A}{2\pi t}\left[\frac{-(e^{j\omega_0 t} + e^{-j\omega_0 t}) + 2}{2}\right] \times 2$$

$$= \frac{A}{\pi t}\left[-\cos\omega_0 t + 1\right]$$

$$x(t) = \frac{A}{\pi t}\left[1 - \cos\omega_0 t\right] \quad \text{(or)} \quad x(t) = \begin{cases} 0 & ; t = 0 \\ \frac{A}{\pi t}\left[1 - \cos\omega_0 t\right] & ; t \ne 0 \end{cases}$$

INVERSE FT FROM SPECTRA



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$$= \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} A \cdot e^{-j\omega t_0} \cdot e^{j\omega t} \cdot d\omega = \frac{A}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega(t-t_0)} \cdot d\omega$$

$$= \frac{A}{2\pi}\left[\frac{e^{j\omega(t-t_0)}}{j(t-t_0)}\right]_{-\omega_0}^{\omega_0} = \frac{A}{2\pi j(t-t_0)}\left[e^{j\omega_0(t-t_0)} - e^{-j\omega_0(t-t_0)}\right]$$

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