FINDING INVERSE FOURIER TRANSFORMS USING PARTIAL FRACTION EXPANSIONS

(*) Find IFT of $X(\omega) = \dfrac{4(j\omega)+6}{(j\omega)^2+6(j\omega)+8}$

$$X(\omega) = \frac{4(j\omega)+6}{(j\omega)^2+6(j\omega)+8} = \frac{4(j\omega)+6}{(j\omega+2)(j\omega+4)} = \frac{A}{(j\omega+2)} + \frac{B}{(j\omega+4)}$$

$$A = \left.\frac{X(\omega)}{(j\omega+4)}\right|_{j\omega=-2} = \left.\frac{4(j\omega)+6}{(j\omega)+4}\right|_{j\omega=-2} = \frac{-8+6}{-2+4} = -1$$

$$B = \left.\frac{X(\omega)}{(j\omega+2)}\right|_{j\omega=-4} = \frac{4(j\omega)+6}{(j\omega)+2} = \frac{-16+6}{-4+2} = 5$$

$$\therefore X(\omega) = -\frac{1}{(2+j\omega)} + 5\frac{1}{(4+j\omega)}$$

Taking IFT on both sides, we get

$$x(t) = FT^{-1}\left[-\frac{1}{2+j\omega}\right] + FT^{-1}\left[5\frac{1}{4+j\omega}\right]$$

We know $e^{-at}u(t) \longleftrightarrow \dfrac{1}{a+j\omega}$.

$$\therefore x(t) = -e^{-2t}u(t) + 5e^{-4t}u(t)$$

(*) Find the IFT of $X(\omega) = \dfrac{1+3(j\omega)}{(3+j\omega)^3}$

Sol:

$$X(\omega) = \frac{1+3(j\omega)}{(3+j\omega)^3} = \frac{A_0}{(3+j\omega)^2} + \frac{A_1}{(3+j\omega)^2} + \cancel{\frac{C}{(3+j\omega)^3}} +$$

for repeated roots

$$A_0 = (3+j\omega)^2 X(\omega)\Big|_{j\omega=-3} = 1+3(j\omega)\Big|_{j\omega=-3} = 1-9 = -8$$

$$A_1 = \frac{1}{1!}\frac{d}{dj\omega}\left[(3+j\omega)^2 \cdot X(\omega)\right]_{j\omega=-3} = \frac{d}{d(j\omega)}\left[1+3(j\omega)\right]_{j\omega=-3} = 3$$

∴ ~~X(jω) =~~

Hint: $X(j\omega) = \dfrac{1}{(j\omega+a)^n} = \dfrac{A_0}{(j\omega+a)^n} + \dfrac{A_1}{(j\omega+a)^{n-1}} + \cdots \dfrac{A_{n-1}}{(j\omega+a)}$

where $A_n = \dfrac{1}{n!}\dfrac{d^n}{d(j\omega)^n}\left[(a+j\omega)^n \cdot X(\omega)\right]_{j\omega=-a}$

$$\therefore\ X(\omega) = \frac{-8}{(3+j\omega)^2} + \frac{3}{(3+j\omega)}$$

Taking IFT

$$x(t) = \text{IFT}\left[\frac{-8}{(3+j\omega)^2}\right] + \text{IFT}\left[\frac{3}{3+j\omega}\right]$$

We know $e^{-at}u(t) \longleftrightarrow \dfrac{1}{a+j\omega}$

$t \cdot e^{-at}u(t) \longleftrightarrow \dfrac{1}{(a+j\omega)^2}$

$$\therefore\ x(t) = -8t \cdot e^{-3t}u(t) + 3 \cdot e^{-3t}u(t)$$

(*) $X(\omega) = \dfrac{-j\omega}{(j\omega)^2 + 3(j\omega) + 2} = \dfrac{A}{(j\omega+1)} + \dfrac{B}{(j\omega+2)}$

Ans: $x(t) = e^{-t}u(t) - 2e^{-2t}u(t)$

* $x(\omega) = \dfrac{5(j\omega)+12}{(j\omega)^2+5(j\omega)+6}$; find IFT

Ans: $x(t) = 3e^{-3t}u(t) + 2e^{-2t}u(t)$

* $x(\omega) = \dfrac{2(j\omega)^2+5(j\omega)-9}{(j\omega+4)(-\omega^2+4(j\omega)+3)}$; find IFT

Ans: $x(t) = e^{-4t}u(t) - 2e^{-t}u(t) + 3e^{-3t}u(t)$

* $x(\omega) = \dfrac{6j\omega+16}{(j\omega)^2+5(j\omega)+6}$; find IFT

Ans: $x(t) = \left(2e^{-3t} + 4e^{-2t}\right)u(t)$

* $x(\omega) = \dfrac{j\omega-2}{-\omega^2+5j\omega+4}$; find IFT

Ans: $x(t) = 2e^{-4t}u(t) - e^{-t}u(t)$

* $x(\omega) = \dfrac{2(j\omega)^2+12j\omega+14}{(j\omega)^2+6j\omega+5} \Rightarrow x^2+6x+5 \overline{)\,2x^2+12x+14\,}\,(2$

$$\begin{array}{r} 2x^2+12x+10 \\ \hline 4 \end{array}$$

$x(\omega) = 2 + \dfrac{4}{(j\omega)^2+6j\omega+5} = 2 + \dfrac{4}{(j\omega+5)(j\omega+1)}$

$= 2 + \dfrac{A}{5+j\omega} + \dfrac{B}{j\omega+1}$

$A = \left.\dfrac{4}{j\omega+1}\right|_{j\omega=-5} = \dfrac{4}{-5+1} = -1$

$B = \left.\dfrac{4}{j\omega+5}\right|_{j\omega=-1} = \dfrac{4}{-1+5} = 1$

$x(\omega) = 2 + -\dfrac{1}{5+j\omega} + \dfrac{1}{j\omega+1}$

Taking IFT

$$\underline{\underline{x(t) = 2\delta(t) - e^{-5t}u(t) + e^{-t}u(t)}}$$

(*) $X(j\omega) = \dfrac{j\omega+3}{(j\omega+1)^2}$; find IFT

Sol:

$$X(j\omega) = \frac{A_0}{(j\omega+1)^2} + \frac{A_1}{(j\omega+1)}$$

where $A_0 = (j\omega+1)^2 X(j\omega)\Big|_{j\omega=-1} = \Big[j\omega+3\Big]_{j\omega=-1} = 2$

$$A_1 = \frac{1}{1!}\frac{d}{d(j\omega)}\Big[(j\omega+1)^2 X(j\omega)\Big]_{j\omega=-1} = \frac{d}{dj\omega}\Big[3+j\omega\Big]_{j\omega=-1}$$

or $\underline{\underline{A_1 = 1}}$

$$\therefore X(j\omega) = \frac{2}{(1+j\omega)^2} + \frac{1}{(1+j\omega)}$$

Taking IFT, we get

$$x(t) = \underline{\underline{2te^{-t}u(t) + e^{-t}u(t)}}$$