Find the IZT of $X(z) = \dfrac{1 - z^{-1} + z^{-2}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})(1-z^{-1})}$; $1 < |z| < 2$

sol:

$$X(z) = \frac{z^3 - z^2 - z}{(z-\frac{1}{2})(z-2)(z-1)}$$

$$\frac{X(z)}{z} = \frac{(z^2 - z - 1)}{(z-\frac{1}{2})(z-2)(z-1)}$$

$$\frac{X(z)}{z} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-2} + \frac{C}{z-1}$$

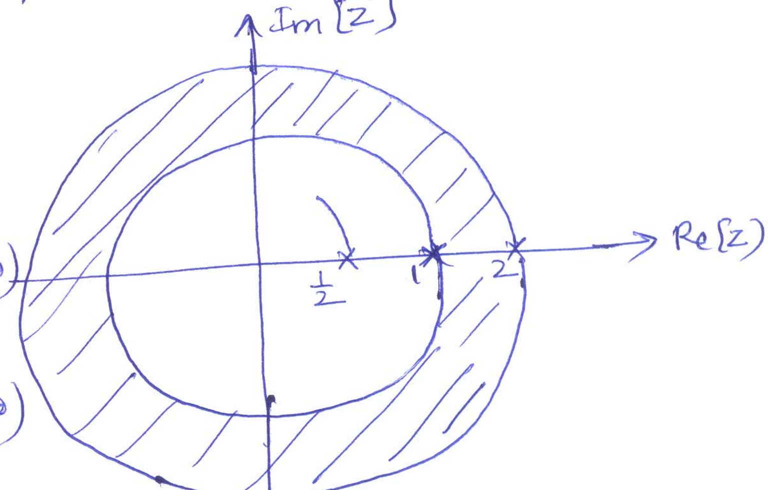
$$\frac{X(z)}{z} = \frac{1}{z-\frac{1}{2}} + \frac{2}{z-2} + \frac{(-2)}{z-1}$$

$$X(z) = \frac{z}{z-\frac{1}{2}} + \frac{2z}{z-2} - \frac{2z}{z-1}$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{2}{1-2z^{-1}} - \frac{2}{1-z^{-1}} \quad \text{--- (1)}$$

poles are $z = \frac{1}{2}$, $z = 1$, $z = 2$.

Given ROC is



∴ (1) pole $z = \frac{1}{2}$ corresponds to causal signal (right sided)

(2) pole $z = 1$ corresponds to causal signal (right sided)

(3) pole $z = 2$ corresponds to anticausal (left-sided) signal

(1) ∴ pole $z=\frac{1}{2}$ corresponds to causal signal (right-sided)

$$\frac{1}{1-\frac{1}{2}z^{-1}} \longleftrightarrow \left(\frac{1}{2}\right)^n u[n].$$

(2) pole $z=1$ corresponds to causal signal (right-sided)

$$\frac{1}{1-z^{-1}} \longleftrightarrow u[n]$$

(3) pole $z=2$ corresponds to anticausal signal (left-sided)

$$\frac{1}{1-2z^{-1}} \longleftrightarrow -(2)^n u[-n-1]$$

∴ The signal $x[n]$ is given by

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - 2(2)^n u[-n-1] - 2u[n]. \quad //$$

Ex:

$$X(z) = \frac{1}{(1+z^{-1})(1-z^{-1})^2}$$; find IZT for a causal signal

Eliminate the −ve powers of z

$$X(z) = \frac{z^3}{(z+1)(z-1)^2}$$

$$\frac{X(z)}{z} = \frac{z^2}{(z+1)(z-1)^2}$$

$$\frac{X(z)}{z} = \frac{A}{(z+1)} + \frac{B_2}{(z-1)^2} + \frac{B_1}{(z-1)}$$

$$A = (z+1)\frac{X(z)}{z}\Big|_{z=-1} = \frac{z^2}{(z-1)^2}\Big|_{z=-1} = \frac{1}{4}$$

$$B_2 = \left[(z-1)^2 \cdot \frac{X(z)}{z}\right]_{z=1} = \left[\frac{z^2}{z+1}\right]_{z=1} = \frac{1}{2}$$

$$B_1 = \frac{1}{1!} \frac{d}{dz}\left[\frac{z^2}{z+1}\right]_{z=1} = \left[\frac{(z+1)(2z) - z^2(1)}{(z+1)^2}\right]_{z=1}$$

$$\Rightarrow B_1 = \frac{3}{4}$$

$$\therefore \frac{X(z)}{z} = \frac{1}{4}\left[\frac{1}{z+1}\right] + \frac{1}{2}\left[\frac{1}{(z-1)^2}\right] + \frac{3}{4}\left[\frac{1}{z-1}\right]$$

$$X(z) = \frac{1}{4}\left[\frac{z}{z+1}\right] + \frac{1}{2}\left[\frac{z}{(z-1)^2}\right] + \frac{3}{4}\left[\frac{z}{z-1}\right]$$

$$X(z) = \frac{1}{4}\left[\frac{1}{1+z^{-1}}\right] + \frac{1}{2}\left[\frac{z^{-1}}{(1-z^{-1})^2}\right] + \frac{3}{4}\left[\frac{1}{1-z^{-1}}\right]$$

We know $a^n u[n] \longleftrightarrow \dfrac{1}{1-az^{-1}}$

$$n \cdot a^n u[n] \longleftrightarrow \frac{az^{-1}}{(1-az^{-1})^2}$$

$$\therefore x(n) = \frac{1}{4}(-1)^n u[n] + \frac{1}{2} \cdot n\, u[n] + \frac{3}{4} u[n]$$

(or) $$x[n] = \left[\frac{1}{4}(-1)^n + \frac{3}{4} + \frac{1}{2} n\right] u[n]$$

Ex: Find the IZT of the following ZT

$$X(z) = \frac{z^3 - 10z^2 - 4z + 4}{2z^2 - 2z - 4} \quad \text{with } |z| < 1$$

Sol: The given rational $X(z)$ is improper as the order of N^r is greater than the order of D^r.

Using long division to express in proper rational function

$$
\begin{array}{r|l}
 & 0.5z - 4.5 \\
\hline
2z^2 - 2z - 4 & z^3 - 10z^2 - 4z + 4 \\
 & z^3 - z^2 - 2z \\
\hline
 & -9z^2 - 2z \\
 & -9z^2 + 9z + 18 \\
\hline
 & 11z - 18
\end{array}
$$

$$\therefore X(z) = 0.5z - 4.5 + \left(\frac{11z - 18}{2z^2 - 2z - 4}\right)$$

let
$$X_1(z) = \frac{11z - 18}{2z^2 - 2z - 4} = \frac{5.5z - 9}{z^2 - z - 2}$$

$$= \frac{5.5z - 9}{(z+1)(z-2)} = \frac{0.5}{z+1} - \frac{6}{z-2}$$

$$\therefore X(z) = 0.5z - 4.5 + \frac{0.5}{z+1} - \frac{6}{z-2}$$

$$X(z) = 0.5z - 4.5 + 0.5\left[\frac{z^{-1}}{1+z^{-1}}\right] - 6\left[\frac{z^{-1}}{1-2z^{-1}}\right]$$

We know

$$\cancel{(-1)^n u[n] \longleftrightarrow \frac{1}{1+z^{-1}}}$$

$$\cancel{(-1)^{n-1} u[n-1] \longleftrightarrow \frac{z^{-1}}{1+z^{-1}}}$$

$$(2)^n u[n] \longleftrightarrow \frac{1}{1-2z^{-1}}$$

$$(2)^{n-1} u[n-1] \longleftrightarrow \frac{z^{-1}}{(1-2z^{-1})}$$

as ROC is $|z|<1$, we need to look for anti-causal signals.

$$-(a)^n u[-n-1] \overset{z}{\longleftrightarrow} \frac{1}{1-az^{-1}}$$

$$\therefore \quad \frac{1}{1+z^{-1}} \longleftrightarrow -(-1)^n u[-n-1]$$

$$\frac{z^{-1}}{1+z^{-1}} \longleftrightarrow -(-1)^{n-1} u[-(n-1)-1]$$

$$= -(-1)^{n-1} u[-n]$$

and

$$\frac{1}{1-2z^{-1}} \longleftrightarrow -(2)^n u[-n-1]$$

$$\frac{z^{-1}}{1-2z^{-1}} \longleftrightarrow -(2)^{n-1} u[-(n-1)-1]$$

$$= -(2)^{n-1} u[-n].$$

$$\therefore \quad x(n) = 0.5\,\delta[n+1] - 4.5\,\delta[n] + 0.5\left[-(-1)^{n-1} u[-n]\right] - 6\left[-(2)^{n-1} u[-n]\right]$$

$$x[n] = 0.5\,\delta[n+1] - 4.5\,\delta[n] - 0.5(-1)^{n-1} u[-n] + 6(2)^{n-1} u[-n] \;//$$

Ex:

$$x(z) = \frac{1}{(z-1)(z-0.5)}$$

Ans: $x[n] = 2\left[1-(0.5)^{n-1}\right]u[n-1]$.